Numeral Systems and Data Structures

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The interrelationship between numerical representations and data structures is efficacious. However, in many write-ups such connection has not been made explicit. As far as we know, their usage was first discussed in the seminar notes by Clancy and Knuth. Early examples of data structures relying on numeral systems include finger search trees and binomial queues. The basic two operations defined on a numeral system are the increment and decrement of arbitrary digits. A numeral system is, therefore, sometimes referred to as a counter.

In this talk we mention some known numeral systems and survey their usage in existing worst-case efficient data structures. We formalize properties of numeral systems and requirements that should be imposed on them to guarantee efficient performance on the corresponding data structures. We start by surveying the regular and extended-regular numeral systems [1]. There, we show how to efficiently implement the increment and decrement operations in constant time using an array [3]. We also discuss two other regular binary systems:

The strictly-regular system [4] is a compact system that supports increments and decrements in constant number of digit flips. The strictly regular system is superior to the regular system for its efficient support of decrements, and superior to the extended-regular system for using three symbols instead of four.

The in-place binary counter [2] is an even more compact system that only supports incrementing and decrementing the least-significant digit in constant time in the RAM model. The space used by the in-place counter to handle the non-negative integers up to $2^n - 1$ is at most $n + O(\lg n)$ bits.

References

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[4] A. Elmasry, C. Jensen and J. Katajainen, Strictly-regular number system and data structures, *12th SWAT* (2010), *LNCS* **6139**, 26–37.